Muller, P. M. and Stephenson, F. R., 1974. The acceleration of the Earth and Moon from early astronomical observations, in *Growth Rhythms and History of the Earth's Rotation*, eds. G. D. Rosenberg and S. K. Runcorn, John Wiley & Sons, London, 459-534.

Muller, P. M. and Van Flandern, T. C., 1977. Determination of the rate of change of G from solar system observations 1375 BC-1975 AD, in preparation.

Oesterwinter, C. and Cohen, C. J., 1972. New orbital elements for Moon and planets, *Celest. Mech. 5*, 317-395.

Van Flandern, T. C., 1970. The Secular Acceleration of the Moon, *Astr. J. 75*, 657-658.

Van Flandern, T. C., 1975. A determination of the rate of change of G, *Mon. Not. R. Astr. Soc. 170*, 333-342.

Van Flandern, T. C., 1976. Is gravity getting weaker? *Scient. Amer. 234*, 44-52.

A Laboratory Measurement of the Constancy of G

by Rogers C. Ritter and Jesse W. Beams

The possibility of a laboratory test of the rate of change of G at the level of 10^{-11} has usually been discounted.[1] Partly this has been due to the historical concentration of laboratory gravitational measurements on the absolute value of G which until recently have been limited in accuracy to 10^{-3} or worse.[2] On the other hand the enormous advancement in recent years of the experimental techniques germane to the problem as well as the fact that a change in G can be determined with much greater accuracy than its absolute value now makes a level of precision of $|\dot{G}/G|$ seem reasonable to expect from an appropriately designed torsion balance. Actually, one or two orders of magnitude improvement on this seem possible, depending on isolation from the hostility of an earthbound environment.

Astronomical or satellite experiments operate in a natural regime for the observation of changes in G. Satellites, however, are not free of problems of their own,[3] and there is no doubt that the accessibility and control of a laboratory experiment, if properly monitored, is a powerful motivation in providing confidence for such a fundamental question.

1. The experiment

In a torsion balance, the gravitational force of attraction must be matched against a restraining force of some kind. Initially it was believed that centrifugal force would be an ideal source of restraining torque. Such an experiment, however, would have a null result regardless of whether *G* is changing if we assume[4] that the local inertial forces are equivalent to gravitational forces due to the presence of distant accelerated matter. In fact, a test of this interpretation of Mach's principle could be made in such an experiment, but it would more logically follow the present one.

The use of an electromagnetic drag, torsion fiber or diamagnetic solenoid would result in comparing GmM with e^2 (we assume c and h are true constants) where m and M are the attracting masses. Present limits of the change in the fine structure constant as determined in the laboratory are of the order of 10^{-12} per year[5] and as deduced from astronomical red shift they are several orders of magnitude better than this. Thus the basic question of changes in e is not presently a problem for this experiment.

Fig. 1 depicts a simplified schematic diagram of the experiment. The entire apparatus rotates at approx-

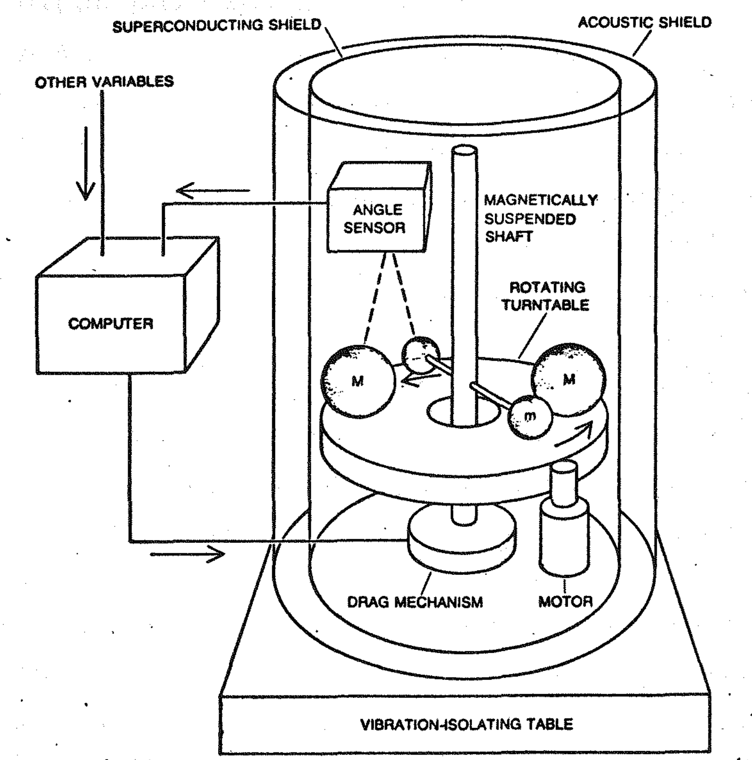


Figure 1. Simplified schematic diagram of the experiment. Experiment. Experimental parameters: large mass separations - 20 cm.; small mass separations - 10 cm.; equilibrium angle - 26°; temperature - 50 mK; pressure - << 10^{-8}mm Hg. See text for details of operation.

imately 3 revolutions per minute to protect against effects of extraneous surrounding masses. The small masses (possibly more in number than shown) are freely suspended and gravitationally attracted forward toward the large masses. The drag mechanism holds the small masses back so that they do not gradually rotate into the large masses because of the mutual gravitational attraction. An angle sensor measures the angle between the small and large masses and feeds that information into a computer, which in turn tells the drag system how much force it must exert to keep the angle constant. The magnitude of that force gives a direct measurement of the gravitational attraction between the small and large masses. Any change in the force will be interpreted as a change in *G*mM, and consequently with near certainty as a change in *G*. (Universal changes in mass are not experimentally ruled out.)

Although an electromagnetic drag mechanism seems at the present time to be a preferred method of restraint, this question is under study.

The classical torsion balance restraint, a fiber, seems to present a host of materials problems. Drift in the fiber equilibrium position, with time, has been a continuing difficulty in the recent *G* measurement at the National Bureau of Standards.[6] In that experiment even the use of carbon fibers, a great improvement, has not completely eliminated drift. In the present experiment cooling a fiber to a low temperature would be expected to greatly reduce drift but much study is needed before a fiber could be considered acceptable at the level of stability needed in this experiment.

Although subsequent experimentation could modify details of the experiment, its parameters have been determined on theoretical grounds. The lever arms s_1 and ℓ_1 will be 5 cm and 10 cm, respectively and the masses M = 5 kg and m = 0.5 kg. With this scale and operating at an equilibrium angle near 26° the gravitational torque exerted on the small masses by the large will be about 0.03 dyne-cm.

A sense of the difficulty of the experiment can be

gained by comparison of the modern experiment most nearly like it--the measurement of the equivalence of inertial and gravitational mass, by Dicke and collaborators.[7] In that experiment a gravitational anomaly of one part in 10^{11} would have given a torque of about 6×10^{-10} dyne-cm. That is the signal they expected to see, if any. The present experiment would have a gravitational torque of .033 dyne-cm. A change of one part in 10^{11} per year of this would be about 3×10^{-13} dyne-cm, or about 2000 times smaller than Dicke's. His signal would have been a time-varying one, with 24-hour period, while ours would be expected to be an essentially linear change over a year's period, or some fraction of a year over which a run was carried out.

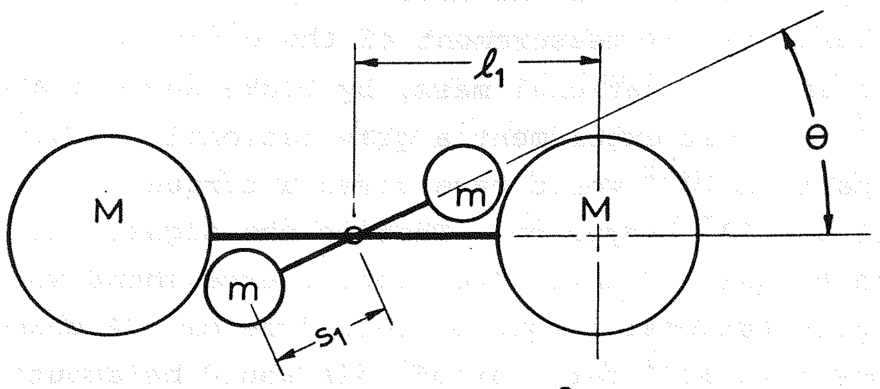
In following sections we will see how these differences dictate certain major changes in design philosophy. For example, the essentially dc character of our expected signal will lead us to treat noise as an intrinsic part of the scaling criteria.

2. *The basic torsion balance*

Basic criteria for sensitivity and noise of the balance can be developed independently of its particular configuration or means of restraint. The object of the analysis of this section is to aid in scaling the balance and in determining its temperature and dynamic properties.

A. *Torque, Stability, and Equilibrium*

The simplest torsion balance is shown in Fig. 2, consisting of large fixed and small (moveable) dumbells. These provide a gravitational mass quadrupole-



$$T_g = f\,F(x,\Theta) \qquad x = \frac{s_1}{\ell_1}$$

$$f = 2GMm\frac{s_1^2}{\ell_1^3}$$

$$F(x,\Theta) = \frac{\sin\Theta}{x}\left\{\frac{1}{(1+x^2-2x\cos\Theta)^{3/2}} - \frac{1}{(1+x^2+2x\cos\Theta)^{3/2}}\right\}$$

Figure 2. Torsion balance configuration showing the factorization of the torque into a scaling part f and an angle-dependent part $F(x,\theta)$.

quadrupole interaction. The torque of this interaction is given by two factors

$$T_g = fF(x,\theta), \qquad \text{[Eq.1]}$$

where f is a multiplying or scaling factor and $F(x,\theta)$ is a factor which depends only on the angle θ and the ratio of the lever arms $x = s_1/\ell_1$.

$$f = \frac{GQq}{2\ell_1^5}, \qquad \text{[Eq.2]}$$

where $Q = 2M\ell_1^2$ is the mass quadrupole moment of the large system, and $q = 2ms_1^2$ is the mass quadrupole moment of the small system. If $\ell_1 = 10$ cm, $x = 0.5$, $m = 0.5$ kg, and $M = 5$ kg, then $f \approx 0.01$ dyne-cm.

The angle factor $F(x,\theta)$ is given by

$$F(x,\theta) = \frac{\sin\theta}{x}\left\{\frac{1}{(1+x^2-2x\cos\theta)^{3/2}} - \frac{1}{(1+x^2+2x\cos\theta)^{3/2}}\right\}. \qquad \text{[Eq.3]}$$

Stability criteria are developed from this by integrating over angle to form a potential function

$$V(x,\theta) = \int_0^\theta F(x,\theta)d\theta = \frac{1}{x^2}\left\{\frac{-1}{(1+x^2-2x\cos\theta)^{1/2}} - \frac{1}{(1+x^2+2x\cos\theta)^{1/2}} + \frac{2}{1-x^2}\right\}. \qquad \text{[Eq.4]}$$

This is plotted in Fig. 3. An angle-independent restraining torque has a constant slope of opposite sign as shown. For example, a soft fiber wound to a large number of turns would have a parabolic shape centered many radians to the right and which very nearly approximates a straight line in this region. The equilibrium angle is that for which the two slopes have equal magnitude.

The s-shaped $V(x,\theta)$ has a point of inflection near 26° in this example, where $x = 0.5$. If the slope V (restraining) is chosen to set the equilibrium to the right of this inflection, it will be a position of unstable equilibrium. As drawn for V (total) the equilibrium point is chosen to the left of the inflection point which gives a slight dip in V (total).

Since V (restraining) is essentially straight, the curvature of V (total) is given by that of $V(x,\theta)$.

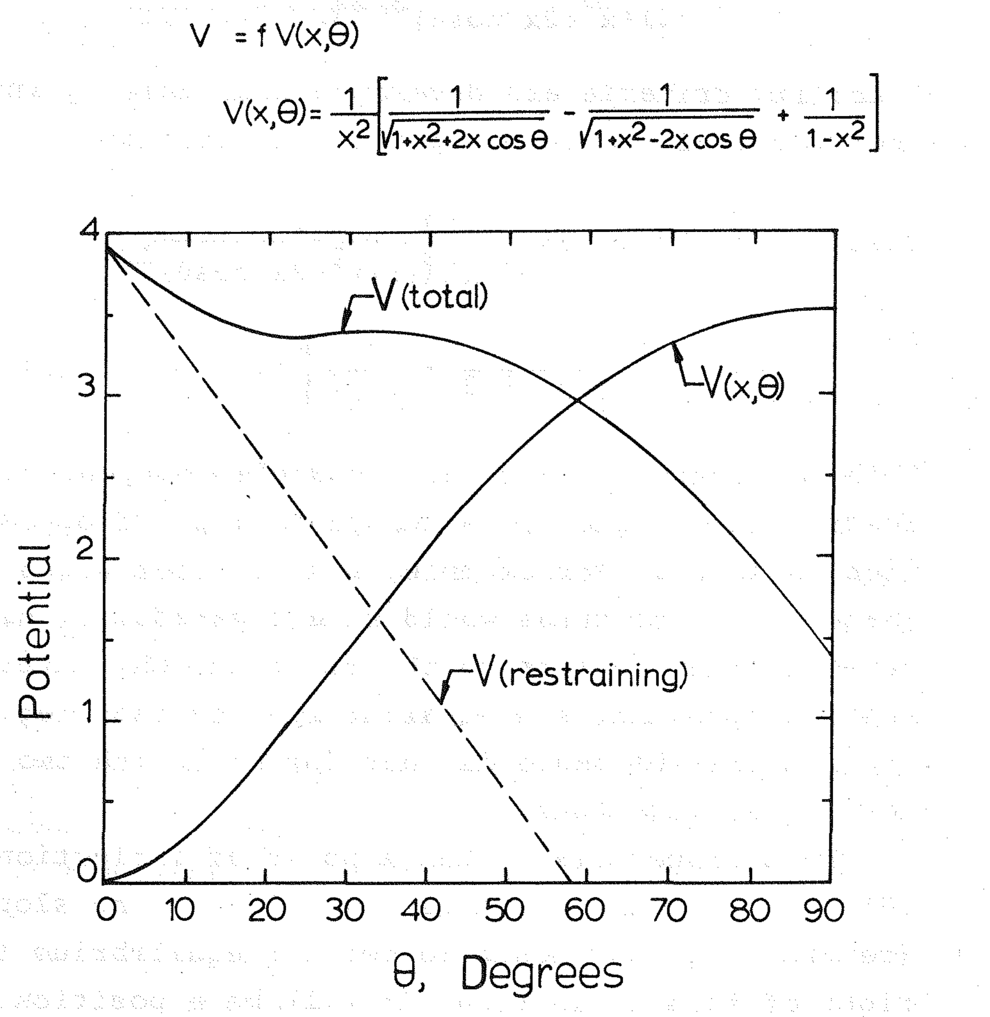


Figure 3. Potential as a function of angle showing the gravitational and (linear) restraining components.

This curvature, $\partial^2 V(x,\theta)/\partial\theta^2 \Big|_{\theta_o} = \partial F(x,\theta)/\partial\theta \Big|_{\theta_o}$, sets the intrinsic sensitivity of the balance. In the following sections, this, when multiplied by the scaling factor f, will become the restoring torque coefficient, C.

B. Noise

Thermal noise of the system gives rise to fluctuations in angle which depend on the temperature T. The noise can be minimized by optimizing damping and feedback as discussed for a torsion pendulum by McCombie.[8] This must interplay with other dynamic problems and we defer the explicit treatment of damping.

An expression for the noise fluctuations independent of the damping is

$$\frac{1}{2} C\overline{\Delta\theta_n^2} = \frac{1}{2} kT, \qquad \text{[Eq.5a]}$$

or

$$\Delta\theta_n = \sqrt{\frac{kT}{C}}, \qquad \text{[Eq.5b]}$$

where k is Bolzman's constant. Here $\Delta\theta_n$ is the rms value of the thermally caused fluctuations in angle.

C. Signal-to-Noise Ratio:

We express the $\dot{G}$ signal in terms of the fractional change in torque during a measurement period

$$\delta = \frac{dT_g}{T_g} \qquad \text{[Eq.6]}$$

as follows.

$$dT_g = \frac{\partial T_g}{\partial \theta} d\theta = C d\theta .$$

$$\therefore d\theta = \frac{T_g \delta}{C} . \qquad \text{[Eq.7]}$$

The signal-to-noise ratio, primitively, is the ratio of [7] to [5b].

$$S/N = \frac{d\theta}{\Delta\theta_n} = \frac{T_g \delta}{\sqrt{CkT}} . \qquad \text{[Eq.8]}$$

D. Signal Averaging

[Eq.8], however, is in terms of the rms value of a randomly varying quantity $\Delta\theta_n$, and the true signal-to-noise ratio can be improved by a sampling method which averages the fluctuations appropriately. This can be taken care of by multiplying [Eq.8] by a signal-averaging factor W:

$$S/N = \frac{T_g \delta}{\sqrt{CkT}} W. \qquad \text{[Eq.9]}$$

The calculation of W depends on the overall data-taking strategy as well as the sampling periods. In effect, we are interested in determining the slope of a straight line through data points which vary randomly because of $\Delta\theta_n$. The points are only independently fluctuating if the sample time is comparable to, or larger than, the effective response time of the instrument. By critically damping the balance and by using negative feedback to decrease the response time to some effective value τ_e, we can increase the number of independent samples in a given run period S. The feedback, as McCombie points out, cannot be used to

reduce noise below optimum but rather is used to improve our signal averaging effectiveness.

W, then should be the reciprocal of the factor by which the uncertainty due to data point fluctuations is reduced by fitting a line through them. The variance σ_m^2 of the slope of a line y = mx + b fit through the points is related to their variance (which we call σ^2) by[9]

$$\sigma_m^2 = \frac{N\sigma^2}{\Delta} = \frac{N\sigma^2}{N\Sigma x_i^2 - (\Sigma x_i)^2} \qquad \text{[Eq.10]}$$

where N is the number of points. If we assume the sample times x_i are spaced equally by the effective response time τ_e, then $x_i = i\,\tau_e/S$, where we have written x_i in dimensionless form. After standard evaluation of the series in the denominator this gives

$$\sigma_m^2 = \frac{12\sigma^2 S^2}{N(N^2-1)\tau_e^2} . \qquad \text{[Eq.11]}$$

Assuming that N >> 1 and $N = S/\tau_e$, this leads to

$$\sigma_m = 2\sigma\sqrt{\frac{3\tau_e}{S}} = 2\sigma\sqrt{\frac{3}{N}} . \qquad \text{[Eq.12]}$$

Therefore, the factor W is

$$W = \frac{\sigma}{\sigma_m} = \frac{1}{2}\sqrt{\frac{S}{3\tau_e}} \qquad \text{[Eq.13]}$$

and the signal-to-noise ratio is

$$S/N = \frac{T_g \delta}{2\sqrt{CkT}} \quad \frac{S}{3\tau_e} . \qquad \text{[Eq.14]}$$

(Alternatively, we could find the intercept of a line fit through the data, which from the above method would lead to the same equation [13] except for omission of the factor $1/\sqrt{3}$.)

E. Sample Calculation

The experiment can be scaled from [Eq.14] as follows: T_g and C are proportional to f, and the other factors are independent of it. (Peculiarities in the ultimate feedback scheme could possibly introduce some dependence of the feedback time constant, τ_e, on f.) Neglecting this we have,

$$S/N \propto \sqrt{f}, \qquad \text{[Eq.15]}$$

so that the larger the experiment, the higher S/N. The dependence is not strong, however, and other factors to be discussed (e.g. effect of extraneous masses) as well as cost favor a smaller experiment. As a compromise we have chosen the values $\ell_1 = 10$ cm, x = 0.5, m = 0.5 kg, and M = 5 kg. This will give a force of .0046 dyne for each pair of masses and a torque of 0.033 dyne-cm.

Having chosen these, we can define a data-taking strategy and determine other corresponding factors in [Eq.14]. As will be discussed later, we expect to run the experiment for a year. It may be too much to expect this to be a single unbroken run and several

two-month periods would be a pessimistic estimate which we will use. For each one S would be two months, or 5.2×10^6 sec.

A value of 10 for S/N would mean that the change in true signal, $d\theta$, would be ten times the standard deviation of the effect of thermal fluctuations in angle on the slope of a line fit through the data. The fractional change in torque, δ, can be evaluated for a two-month run based upon the assumption that $\dot{G}/G = 10^{-11}$ per year. Then $\delta \approx 2 \times 10^{-12}$.

The S/N ratio can be expressed in terms of the moment of inertia, I, of the small mass system and the time constant of the balance, τ, rather than C from

$$\tau = 2\pi \sqrt{I/C} \qquad \text{[Eq.16]}$$

Then [Eq.14] becomes

$$S/N = \frac{T_g \ \delta\tau}{4\pi \sqrt{IkT}} \sqrt{\frac{S}{3\tau_e}} . \qquad \text{[Eq.17]}$$

As scaled above, the small mass system would have a moment of inertia I = 25,000 in cgs units.

These factors, inserted into [Eq.17] lead to a working relation

$$S/N = 3.72 \times 10^{-6} G \sqrt{\frac{\tau_e}{T}} , \qquad \text{[Eq.18]}$$

where G is a feedback factor,

$$G = \sqrt{G_c G_I} , \qquad \text{[Eq.19]}$$

G_C is the direct feedback and G_I is the inertial feedback.

Using the value of 10 for S/N, we arrive at a relation between the temperature T and the time constants:

$$G\frac{\tau}{T} = G^2\frac{\tau_e}{T} = 7.2 \times 10^{12} \, . \qquad \text{[Eq.20]}$$

From this, assuming a temperature of 50 mK, we have $\tau_e = 36$ sec. and $\tau = 42$ days, if $G = 10^5$.

To minimize noise and also sampling time τ_e the system will be critically damped. This implies a damping time $\tau_d = \tau/4\pi = 3.3$ days. To reach this, the damping coefficient κ (see following section) must not exceed 0.087 in cgs units. Such a low damping, with primary magnetic suspension, is possible. If the total restraining torque consisted of a drag mechanism which contributed this much damping the table would have to run at 3.6 rpm.

From the above conditions we can calculate the signal angle, $d\theta$, and the torsion coefficient, C. The signal-averaging factor W takes the value 110 under these conditions [Eq.13]. This leads to $C = 7.6 \times 10^{-8}$ dyne-cm/radian and $d\theta = 8.7 \times 10^{-7}$ radians.

3. Disturbances and experimental design problems.

For convenience the sources of disturbance or drift are categorized (with some arbitrariness) as "internal" or "external."

The special ways in which these affect the experiment will be taken up briefly, along with a few of the design problems peculiar to this experiment.

Sources of Disturbances or Drifts

Internal	*External*
1. Thermal Variations	1. Magnetic Field Variations
2. Mass Stability	2. Surrounding Mass Variations
3. Dimensional Stability	3. Ground Vibrations
4. Angle-Sensor Stability	4. Stability of Angular Velocity
5. Electrostatic Variations	5. Miscellaneous Geo-physical Effects
6. Pressure Variations	
7. Restraint Stability	

A. Internal Sources

1. Thermal variations - The most obvious effects of thermal variations are on the mass, dimensional, angle-sensor and restraint stabilities and on residual gas pressure variations. Discussion of these are deferred to individual sections. We note here that an operating temperature of ∿50 mK has been chosen.

A subtle effect has to do with changes in symmetry of the balance. It will be shown that multiple small masses in symmetric arrangements can be important in reducing effects of vibrations and in monitoring the experiment. This symmetry is expected to be achieved at a level of a part in 10^6. If it is to be useful, it must not change from the situation in which it is set up and tested to the situation during the experiment. Operation at ∿50 mK would be expected not to create gradients such as to cause asymmetry at this level. However, internal tests for symmetry will be built into the experiment.

It is clear that heat leaks and the ^{3}He refrigeration must be kept as constant as possible during a run. Details of the design, including possible heat sources from angle-sensing, are important in any estimate of the temperature variations in this experiment. In succeeding estimates we will use a very conservative figure of $\Delta T = 25$ mK.

2. *Mass stability* - The absolute mass of the International Kilogram is only known[10] to be constant to about 5 parts in 10^9. Such measurements, however, involve a change of atmosphere for the mass, and its cleaning. The stability of a cooled mass in constant environment, such as in the proposed experiment, is a different problem.

It is well known that even in a high vacuum a metal will give off or absorb gas or vapor and that chemical reaction will occur, especially on its surfaces. Such effects would give rise to changes in the masses of an amount which might not be tolerated. Fortunately in the measurements of changes in *G* it is only necessary to keep the masses constant and not essential to know their absolute values with high precision. This permits some freedom in the shapes of the masses and also in their materials.

Experiments of J. W. Mitchell, at the University of Virginia, and others have shown that a carefully deposited gold coating on some metals and on glass surfaces not only is free of observable surface reaction but also seals the surface from observable gas or vapor transmission even at room temperature.

At 1° K and lower, vapor pressures of all substances except helium and hydrogen are for our purposes negligibly low. Consequently, it will be necessary to prevent the adsorption and emission principally of helium from the surfaces of the masses M and m where the pressure of helium in the vacuum will not exceed 10^{-6} torr. (We actually expect the pressure to be

very much lower than this.) A calculation shows that the change in mass due to the exchange of helium with M and m would be too small to give trouble especially if the temperature gradients are not allowed to become both excessive and variable.

3. *Dimensional stability* - Even at constant temperature, materials exhibit some dimensional change. This occurs particularly under a variation of mechanical stress, and the design of this experiment aims at minimizing such variation. If it is accepted that the linear coefficient of thermal expansion approaches zero at zero temperature an estimate of an upper limit on expansion can be made by a crude extrapolation. For example, 304 steel has a coefficient $1/L\ (dL/dT)$ of 10^{-8} at 10° K. At this point the temperature dependence is changing from the T^3 law to something of lower power. A linear extrapolation to 50 mK will, nevertheless, overestimate the true value of the expansion coefficient. Carried out, this leads to a value of 10^{-12} change in L for a temperature change of 25 mK. Other, more favorable materials can be found with a lower coefficient.

4. *Angle-sensor stability* - The signal angle, $d\theta$, for a two-month run, assuming $\dot{G}/G = 10^{-11}$/yr was calculated to be 8.7×10^{-7} radians. The angle sensor should be sensitive and stable to at least an order of magnitude better than this or $\approx 10^{-7}$ radians. (On a 5-cm arm, this corresponds to a distance of 5×10^{-7} cm.) Such an angle is not difficult to measure, even by ordinary optical means. However, since feedback of the signal has a complex nature, the sensor must meet requirements besides sensitivity. Noise and drift appear as directly additive to the signal. Derivative

feedback enhances the effect of noise on the output reading and loop correction, while integral feedback increases the effect of drift. One can conclude that stability and noise should be far below the level of 10^{-6} radians.

The two angle-sensing methods being considered most seriously both meet these criteria. Laser interferometry can be this sensitive and stable providing something like Fabry-Perot optics are used. Alternatively, a SQUID-sensed proximity method has been used with extreme sensitivity.[11] A strong feature of the laser method is the flexibility of optical paths. For example, the Sagnac effect could be used to provide angle-sensing independent of translational motion of the masses. A strong feature of the SQUID method is the low noise and the fact the experiment is already at superconducting temperatures.

A factor to be considered is the contribution of the sensor to the forces present. For example, both photon fluctuations and drift of light intensity could create problems. If laser interferometry is to be used, the intensity fluctuations would be a problem, even with intensity stabilization, at the level of sensitivity of this experiment. The use of beam-splitting, sending beams to opposing members of a symmetrized system relieves the problem somewhat, but it introduces the need for consideration of stability of reflectance and absorption at each surface.

A sample calculation indicates the nature of the problem of noise from photon pressure fluctuations in a laser-sensor. For $\lambda = 10^4$ Å, the energy per photon is about 2×10^{-12} erg and the momentum is about 6.6×10^{-23} gm-cm/sec. The gravitational force on each small

mass is .0033 dynes in our system, and 10^{-12} of this is 3.3×10^{-15} dynes. This value of force fluctuation would be caused by $\delta N = 5 \times 10^7$ photons. If an extreme (statistical) relationship is assumed between photons, it implies a total number $N = (\delta N)^2 = (5 \times 36 \times 10^7)^2 = 3.2 \times 10^{18}$ photons in a period during which the system can respond to the fluctuations, assumed here to be about 36 seconds. This corresponds to a beam power of about 18 milliwatts. In a split beam system in which coherence in fluctuations is not maintained between the split components there is no gain from symmetrization. Actual laser beam fluctuations, however, have non-statistical components which could be made to cancel with such symmetry, with appropriate attention to design. In summary, the statistical part of laser beam fluctuations could in principle limit the beam power to something below 18 milliwatts--well above thermal limitations.

Laser interferometry, without special optical methods, could be position-sensitive to a small fraction of a wavelength. With Fabry-Perot optics and special methods, it may be capable of sensing to 10^{-12} cm.[12] This sensitivity leaves a large margin and concomitantly offers good noise and drift properties. The finesse, which could be as high as 500 to 1000, provides noise-free gain for the sensor, which is important in lowering its effective temperature to maintain optimum feedback and damping conditions. A further advantage is the linearity, which can be sustained over many orders of magnitude of signal change.

A type of position sensor which has been suggested to us by Fairbank uses a SQUID magnetometer. This senses the magnetic field of a persistent current in a loop on the large mass, modified by the proximity of a superconducting coating on the small mass. Under certain conditions,[13] positional sensitivity to 10^{-14} cm is expected. With the present dc application, studies of drift and noise are needed. The noise of a SQUID is still not determined with certainty, and it depends on the magnetic field noise which is present. In measurements of static magnetization[14] the device temperature has been found to be $\lesssim$ 1 mK, with some implication that it is unmeasurably low. Several versions of this method are used in gravitational experiments at Stanford,[15] and the experience gained there will be valuable in assessing its potential for the present experiment.

Both methods of angle sensing have the potential for meeting our need. They both require a high degree of technical competence in order to adapt them appropriately to this experiment. It is not certain how long they can operate free of fault. And, finally, it is not clear which would be easiest to adapt to our symmetrical multiple-mass configuration. It is our intention to do a preliminary experiment combining and comparing the two methods to gain experience with them and to assess the appropriateness of their particular properties.

5. *Electrostatic variations* - The accumulation or change of free charge on an insulated or nongrounded surface such as might be produced by cosmic radiation is the most obvious effect. It can be eliminated by designing the system so that all essential surfaces are conducting and grounding them.

A second effect is due to contact potential differences from different materials. If these are taken to

be about 0.5 volt for surfaces of 100 cm^2 separated by one cm, a force of about 10^{-4} dyne will result. The gravitational force is about 0.0046 dyne, or only 50 times greater. Thus changes in the electrostatic force must be kept to roughly a part in 10^{10} over any run.

To date some of the best experience with electrostatic effects on superconducting surfaces has been by Fairbank et al.[16] The relative effects of electric and gravitational fields on the electron (and positron) were studied, and indeed were crucial to the success of their experiments. They were able to distinguish effects on electron flights due to changes of 5 x 10^{-11} V/m in the presence of whatever disturbing surface effects were present (e.g. the "patch effect"). This is not quite a direct result for us, but it implies that, over short time periods, surface effects may be reducible to less than 10^{-12} volts per cm at such pressures and temperatures.

6. *Pressure variations* - Pressure variations can affect the experiment in several ways. Two are considered here: 1) the force differences imposed on the sides of the small masses, and 2) variations in the viscous drag.

The first effect has two categories: convective problems and fluctuations. The convective pressure differences must be made small by having constant, uniform temperature. An estimate of the magnitude of such effects can be gotten by calculating forces due to some pressure difference ΔP across the opposite sides of the small masses, say spheres of 1.83-cm radius. A pressure difference of 2.4 x 10^{-7} mmHg would be sufficient to cause a torque equal to the gravitational torque, T_g = 0.033 dyne-cm. An acceptable value, 10^{-12} of this, would be 2.4 x 10^{-19} mmHg, truly a small pressure difference. Great care will be needed in the design to eliminate heat sources and to keep the ambient pressure low so this can be maintained. Although we expect the pressure to be lower, we will calculate with a conservative value of 10^{-8} mmHg.

Fluctuations of pressure are intimately connected with damping because viscous drag due to ambient pressure provides part of the intrinsic damping in this experiment, along with the drag of the restraining torque.

In 36 seconds (the assumed effective time constant of the balance) at 10^{-8} mmHg pressure and T = 50 mK, there are N = 4.3 x 10^{17} collisions on a side of the sphere. These would correspond to pressure fluctuations on that side of 1.5 x 10^{-17} mmHg. This is nearly 100 times greater than the acceptable value estimated above. However, the pressure-induced motional fluctuations are reduced (along with the effective temperature) by the application of derivative feedback.

The magnitude of the viscous damping of each small mass can be estimated. At 10^{-8} mmHg and 50 mK the molecular mean free path (assuming a helium atmosphere) is about 2 meters, so the free-molecule regime is applicable. An equation[17] for the drag force D when specular reflection is assumed for collisions on a sphere of radius a is

$$D_{spec} = nkT\pi a^2 \left[\frac{1}{\sqrt{\pi}}(V^{-1} + 2V)e^{-V^2} + (2V^2 + 2 - \frac{V^{-2}}{2})\,\mathrm{erf}V \right], \qquad \text{[Eq.21]}$$

where n is the molecular number density, $V = u/\sqrt{2kT/m}$ is the "reduced velocity," m is the molecular mass and u is the velocity of the sphere through the gas. For the case encountered here $V << 1$, and [Eq.21] becomes approximately

$$D_{spec} \approx nkT \sqrt{\pi}\, a^2\, 16V/3.$$

For diffuse reflection of molecules against the sphere this is multiplied by 1.39. Using this, and assuming a = 1.83 cm for the present experiment, we find $D_{diff} \approx 4.1 \times 10^{-7}$ u in cgs units.

Applied to the homogeneous second-order differential equation of motion, $I\ddot{\theta} + \kappa\dot{\theta} + C\theta = 0$, this gives us the damping coefficient $\kappa = 1.0 \times 10^{-5}$ in cgs units.

We intend to operate at the critically damped point for reasons discussed earlier concerning noise and sampling time. The damping coefficient in this case would be $\kappa_c = 0.087$. Thus derivative feedback of gain $\approx 10^4$ can be used, which has the effect of reducing the effective temperature. From McCombie[8] this would lead to an effective temperature

$$T_e = \frac{\kappa}{\kappa + \kappa_c} T \approx 10^{-4} T , \qquad \text{[Eq.22]}$$

where T = 50 mK is the actual temperature.

Such low damping and such low effective temperature are certainly at the frontier of application, though the techniques used to achieve them are not unusual.

B. External Sources

1. *Magnetic field variations* - A difficulty which Dicke and apparently Eotvos encountered was the torque caused by magnetic contaminants. The presence of such contaminants led to torques in the earth's field

which were comparable to the torques expected for the gravitational anomaly. These were made small by efforts to use materials freer and freer of such contaminants.

In the present experiment, operation at superconducting temperatures modifies this situation and rotation further affects it. A superconducting shield around the experiment greatly attenuates the earth's field. It is not uncommon laboratory practice with such a shield to be able to reduce the external field, say one gauss, to a microgauss inside the shield.

Flux that does not change magnitude or direction throughout the experimental volume will not affect the experiment if the masses and their arms do not have changing magnetic moments. Superconducting coatings on these elements, necessary for other purposes, will essentially contain moments due to any such materials contaminations within them, so that they will not interact with constant fields that are present.

Time-varying fields can be present to a small extent within even a perfect superconducting shield even though the total trapped flux does not change. Varying external fields induce currents in the shield which can move the internal flux lines and modify the coupling to any externally manifested moments of the masses.

The greatest time variation of external fields as experienced by the balance is expected to be due to rotation of the complete experiment, as will be discussed in the following section. An intrinsic factor of $\sim 10^9$ reduction is caused by the difference in rotation period and intrinsic balance period, as will be shown. Thus the major concern is that magnetic leakage torques, although averaged out by such an effect,

not have instantaneous values so large as to be rectified by nonlinearities of the balance and measuring system.

2. *Surrounding mass variations* - In torsion balance experiments, the interaction of extraneous masses with the moveable masses has been a common problem. It is said to have caused Eotvos to locate his experiment on ice on an open lake and to run up for observation and leave in a time which was short compared with the balance time constant. It caused Dicke to use an octupole mass configuration with its attendant faster interaction fall-off in distance. Seasonal ground waters (apparently) caused a puzzling variation which limited the accuracy of Heyl's measurements of *G* at the National Bureau of Standards.

In addition to "local" anomalous masses, such as those listed by Dicke,[7] tidal effects give varying gravitational gradients which also affect the balance.

For discussion we will consider the anomalous torque, T_a, exerted by some extraneous mass m on our small mass quadrupole, and located a distance r away. Apart from angle-dependent factors this has the magnitude

$$T_a = 3\ G\ m\ q/r^3, \qquad \text{[Eq.23]}$$

where $q = 2ms_1{}^2$ is the mass quadrupole moment of the small mass system, as before. The ratio of this torque to that exerted by the large masses of the experiment is

$$\frac{T_a}{T_g} = \frac{m}{2M}\,\frac{\ell_1^3}{r^3}\ . \qquad \text{[Eq.24]}$$

With the experiment scaled as in our previous calculation, this implies that a 100-kg man at 2100 meters would cause a value of 10^{-12} for this ratio. Such a situation is intolerable and is one of the main reasons for rotating the experiment.

When the experiment is rotating, the small masses experience an alternating torque at twice the frequency of rotation ω_r (say at twice 3 rpm). The system has a natural frequency much lower than this; for example from [Eq.16] we get $\omega_0 = 1.75 \times 10^{-6}$ radians per second.

A system of resonant frequency ω_0, driven at another frequency ω, responds with an amplitude reduced by the factor

$$\frac{A}{A_0} \approx \left(\frac{\omega_0}{\omega}\right)^2 \frac{1}{Q}, \qquad \text{[Eq.25]}$$

where Q is 1/2 (for critical damping) and $\omega >> \omega_0$. The ratio of resonant amplitude A_0 to amplitude of angle change caused by static torque T_a is just Q, so

$$\frac{A}{A_{static}} \approx \frac{\omega_0^2}{\omega^2}. \qquad \text{[Eq.26]}$$

For $\omega = .064$ rad/sec and $\omega_0 = 1.75 \times 10^{-6}$ this ratio is 1.3×10^9.

In effect, the rotation of the system at a frequency far above the natural frequency of the balance leads to a "shielding factor" against external torques of 10^9. Now the 100 kg man will introduce a fluctuation 10^{-12} of the signal angle $d\theta$ (for $\dot{G}/G = 10^{-11}$/yr) when he is 2 meters from the experiment.

The effect of the man's motion, or the motion of any other extraneous mass, is to modulate the alternating torque experienced by the small mass system. Since the system is broadband (low Q), care must be taken to reduce nonlinearities, which could detect such modulation. This constitutes an important requirement on the design of the experiment.

3. *Ground vibrations* - A most serious difficulty with the torsion balance is its vulnerability to vibrations. Sensitive gravity wave detectors operate with high Q at a frequency $\cong$ 1 kHz. Thus mechanical filtering can be made quite effective in those experiments.

The present experiment must respond at essentially dc, so high-pass filtering with cutoff considerably below 1 Hz is needed. Signal averaging, described previously, performs some of that function, subject to the condition that signal excursions, before averaging, remain within the linear range of the balance.

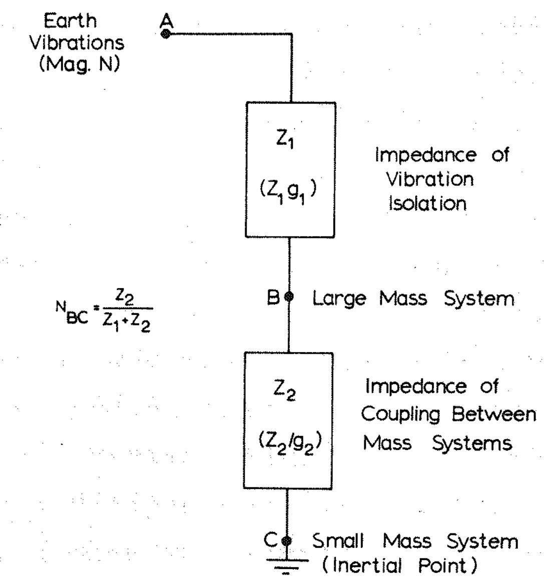


Figure 4. Electrical circuit equivalent to the main vibration properties of the experiment.

The nature of introduction of vibrations into relative motion of the large and small mass systems can be seen in Fig. 4, which is an electrical circuit analog. Due to the high sensitivity needed in the balance, the coupling between the large and small mass systems is extremely weak, represented by a high value of the impedance Z_2. Isolation from ground (the driving point) and the large mass system will be made as effective as possible but cannot be expected to lead to $Z_1 >> Z_2$. Hence, an appreciable fraction of the driving vibrations are across the B C part of the circuit in the passive "voltage divider." Negative feedback in the balance can reduce Z_2 by the factor g_2, which will be complex since direct, derivative, and inertial feedback will all be employed.

In addition a servoed support table for the large masses can increase the impedance Z_1 by some factor g_1. One such table, employed by Sakuma[18] at the International Bureau of Weights and Measures, Sevre, is capable of reduction of the vertical component of vibrational amplitude by a factor of 30. In his system a "double sandwich" table has three plates separated by two sets of stacked piezoelectric transducers. The upper set puts out a signal which is integrated twice, amplified, and fed back out of phase to the lower set of transducers.

We will employ mechanical filtering and a servo table. It is clear that, in view of the complexity of g_1, g_2, Z_1, and Z_2, an overall system design is needed. This can use part of the digital computer to perform the appropriate feedback operations for both the servo table and the balance itself. Monitoring of the servo table corrections will provide information

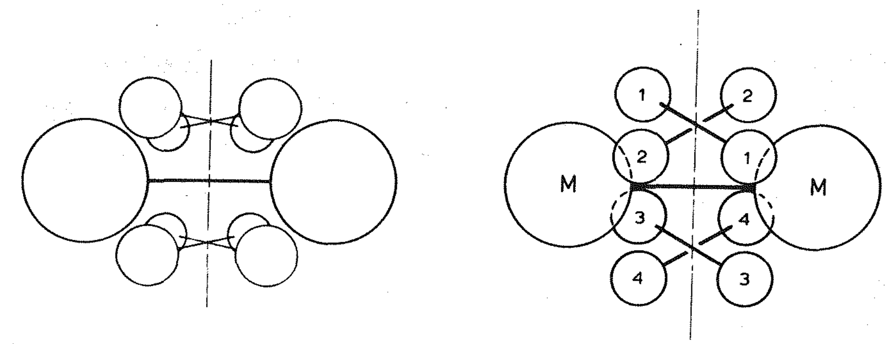


Figure 5. Two drawings of a highly symmetrical system.

about ground vibrations and geophysical effects which, conceivably, could impinge on the signal from the balance.

With appropriate treatment of the signals, several small masses can be used in one of various symmetrical arrangements to cancel effects of vibrations. Fig. 5 shows our maximal present plan, in which eight small masses are symmetrically located about the two large masses. The axis of support and hence the plane of each small mass dumbell is the only physical mechanism which destroys symmetry about the axis of large masses. Otherwise the system would automatically be capable of self-correcting in all directions for vibrations of the large mass. The summation of signals 1 and 2 (or 3 and 4) cancel, to first order, motion of the large masses while adding gravitational effects. The use of four rather than two small mass dumbells adds the ability to correct for vibrations about an axis lying in a horizontal plane. Thus, overall this symmetry corrects for translation of the large masses in three orthogonal directions and, with appropriate signal treatment, vibrational rotations about all axes.

Correction by a factor of 10^6 is possible if the absolute dimensional and mass symmetries are maintained at this level. Whatever initial masses and lengths are achieved must remain that way to a part in 10^{12}, the level needed for success in the stability criterion.

Metrology seems capable of achieving arm-length symmetries to a part in 10^6, but centering the small masses this well is somewhat more difficult. Experience with the sintered tungsten masses in our experiment to measure *G* indicates that density variations lead to uncertainties at the 10^{-5} level in locating the mass center of those spheres.

The use of large single crystals of certain materials might be feasible, and it would certainly improve the prospects of accurately locating the centers of the masses. However, the choice of materials would be reduced and perhaps some sacrifice in density would be necessary. Most likely these masses would be very expensive.

An alternative method which seems more favorable is to balance the masses dynamically by some method. One that seems feasible is to implant or plate material as needed on the small masses to achieve balance when they are supported in the final arrangement. As a practical matter this would probably need to be done before cooling, however. With appropriate materials the change upon cooling need not create differential effects among the lever arms at the 10^{-6} level.

If the centers of the small masses m are known to 10^{-5}, then some of the small mass quadrupoles, $q = 2\ ms_1^2$ would possibly need mass addition up to 2×10^{-5} m to correct for this. For m = 500 gm this

would be 10 mg of material to be placed on the masses. An attractive method would be to use evaporation while at the same time, or alternately, providing a dynamic test of the result.

The location of 10 mg on the mass m would have to be spherically symmetric to correct q ideally, but the empirical method would seem to circumvent that. If the method of test were dynamic, i.e. a comparison of the moments of inertia of all small mass systems by comparing the response of the four signals to an appropriate impulse, the method should correct for vibrations by a factor 10^6. Balance to 10^{-6} by this method would not result in balance of the gravitational torques to that extent if the mass depositions were not perfectly spherical. The importance of such an imbalance would be negligible except for its effect on the method of signal testing.

The complexity and inelegance of this multiple-mass design is disquieting. After some preliminary experiments we may find that a simpler, lower symmetry is sufficient for the final design. Nevertheless, the value of symmetry in dealing with vibrations and for monitoring still seems to be very high.

In summary, integration of the two feedback loops into a common control and monitoring system seems essential. The use of symmetry for further reduction of the effects of vibration truly makes the experiment possible in a reasonable environment.

4. Stability of angular velocity - The complete torsion balance and magnetic shield must be rotated to reduce effects from external masses and also some effects of very low-frequency ground vibrations. This rotation is most effective in an overall sense at a frequency in the vicinity of 3 rpm. This is high enough to provide isolation described above and low enough to avoid undesirable centrifugal effects.

Clearly, angular velocity variations of the driven table move the large masses relative to the (almost) inertially rotating small ones and give rise to a direct variation in signal. The literature gives little information about short-term and long-term accuracy achieved for rotations at ~ 3 rpm. An experiment at the University of Virginia has been carried out as a feasibility study for solution of this problem.[19]

Several effects might be expected from unconstant rotation of the table supporting the large masses. Long-term variations, defined as those having a period comparable to that of the balance, would have two of these effects. Directly, the large masses (which move with the rotating coordinate system with respect to which θ of the differential equation is measured) would forge ahead and backward with the driving variations, contributing directly and to first order to an error in the output signal. If this is to be kept commensurate with a 10 to 1 S/N ratio, such fluctuations in angular velocity could not exceed several parts in 10^{13} if feedback were not used. They occur within the feedback loop, however, and their effects on S/N are reduced by a complex factor.

A second effect of the long-term variations is the coupling via $\dot{\theta}$ and $\ddot{\theta}$ terms into the differential motion of the small masses with respect to the large masses. This in effect introduces fluctuations of the small masses into the balance within the feedback loop. Fortunately these are also reduced by feedback.

Short-term variations will couple negligibly into the small mass system but will give the same direct effect on the signal as the long-term variations. Feedback gives very little correction on these variations.

A fourth effect, which would be troublesome mainly with short-term variations, comes from excursions of the signal beyond the linear range of the system. Rectification would occur and consequently a shift introduced which would appear as a signal.

In Gillies' experiments[19] the long-term variations have been studied with an air bearing turntable driven by a stepping motor fed by our best presently available local clock, having an accuracy of 10^{-10} per day. Rotation periods are measured by a mirror and optical lever system similar to that used in our *G* measurements.[2] The timing and instantaneous driving positional error is 2-4 μsec, and by averaging over successive multiples of tens of periods this contribution can be brought down linearly.

Load variations observed in coast down measurements of the bearing contribute to fluctuations ten times larger than the timing error, so in an unfedback system we were forced to use rather "hard" coupling between the motor and table. This prevented the use of mechanical filtering with long time constants to average out fluctuations in the stepping motor in the frequency range below 1 Hz. As a result, errors of about 0.001 step or 5×10^{-6} revolution could not be eliminated in these measurements.

Results of the measurement are that rms fluctuations of 2×10^{-10} are observed in the averages of 1000 periods, or 4000 seconds in typical runs. Extrapolated to the 3.6×10^6 second period of the balance this would easily provide the needed long-term rotational stability. Whether new effects, e.g. precessional coupling from vibrations, will appear in the next few decades of precision can only be measured when we have a better clock to drive and time the system.

Short-term fluctuations can only crudely be implied from this experiment. The fact that the noise scaled linearly over the period averaging implies a white character to the noise, but extrapolation to sub-periods is highly questionable.

Further study is needed, but present ideas about the design are the following. The rotating system will be magnetically supported. This will reduce load variations below those of an air bearing, but possibly not below the level needed in this experiment. A stepping motor drive, fed from an atomic clock (which also serves as the overall clock for "atomic time" for this experiment) will maintain long-term accuracy. Mechanical filtering will damp out fluctuations with components ≥ 1 Hz, and a feedback system based upon variable drag (e.g. a hysteresis brake) can be tailored to provide corrections in the frequency range in between. Depending on the load variations, these corrections are expected to reduce short-term speed variation by $\sim 10^3$.

5. *Miscellaneous geophysical effects.*[20] - Suppose we examine the effect of variations of the angular velocity of the earth, $\Delta\omega$, the variations of the gravitational field of the earth, Δg, and the wobble of the earth's axis, etc. on this experiment. First, we list some of the values of these quantities as far as they are known:

a) Earth tides: These are produced in the "solid earth" twice daily by the moon and the sun. The change in Δg due to the moon is 0.168 milligal and that due to the sun is 0.075 milligal. (One milligal is about a part in 10^6 of g.) The maximum elevation of the geoid is 36 cm, and the maximum depression is 18 cm due to the moon. The solar tide $\approx$ 25 cm. The combined total at new and full moon is 79 cm.

b) Changes in the earth's rotation are of three types:

1) Secular: decreasing continuously at the rate of 0.0016 sec/century. This decrease is thought to result from tidal friction. It is small enough to be neglected in the $\dot{G}$ experiments.

2,3) Periodic and/or irregular variations: The earth rotation is fastest in August and slowest in March. The difference in length of day (denoted in the literature by l.o.d.) in this period is 2.5×10^{-3} (2.5 milliseconds). If one assumes that this takes place at a uniform rate for 100 days, it would give .0025 sec/100 days = 2.5×10^{-5} sec/day $\simeq 10^{-10}$ l.o.d./day. However, superimposed upon the above must be some irregular variations for as much as 0.0034 l.o.d. is often observed. It is not known what the above irregularities are due to, but they may be due to earthquakes, coupling of mantel and crust, winds, ocean currents. The measurements are deduced principally from the observed motion of the moon. There are, of course, mechanical vibrations of much larger amplitude induced by storms, local activity, etc., but these can be dealt with by damping (effective) in the mount. Fortunately the horizontal table is most helpful with the horizontal oscillations since the vertical components of the motion are relatively less troublesome.

Fortunately if the rotating table is horizontal and driven with a motor fixed to the earth, all of the changes in the rotation of the earth would have *no* effect on the experiment *except* when centrifugal force was used to balance the gravitational interaction. This arises from the fact that the centrifugal force reference is the so-called fixed stars (relativity theory). However, if the rotating table is servoed to the instantaneous-vertical as defined by "gravity" (which may vary with time), then changes in the rotation of the earth do not effect the results even when centrifugal force is used. Also, the wobble of the earth is eliminated. This "wobble" of the earth is manifested by the wandering of the rotation of the earth's axis of rotation. The pole of the earth wanders in roughly an elliptical path with a mean diameter of 20 ft. with a period of about 1 year, where 1.01 ft. corresponds to an angular displacement of 0.010 seconds of arc.

In view of the above, one can conclude that the errors creeping into the experiment due to the *normal* earth motions can be eliminated first because they are mostly periodic and known with sufficient accuracy from astronomical observations and second, they can be greatly suppressed by a servo system which keeps the axis of rotation of the table vertical. (There are very sensitive methods of determining the gravitational vertical.)

C. Miscellaneous design questions

1. Feedback analysis and system design - The feedback system in this experiment has unusual requirements.

The use of direct, derivative and integral feedback in the balance is not an uncommon application, but the time scale is. Only digital methods can accurately handle this when the system time constant is so long.

At the same time, complex feedback will be used in the active vibration isolation table, as part of the overall defense against vibrations. Heavy mechanical filtering can function down to the vicinity of 1 Hz, and the rotation of the system handles some aspect of very low-frequency extraneous motions. Still, the overall analysis of this and integration into a combined system for the balance, for the vibration isolation and for internal monitoring to be discussed later presents a problem in analysis and design which, though not unprecedented in difficulty, is nevertheless unusual. This is especially so considering the need for error-free operation of the dedicated computer in those aspects which are active in continued operation of the experiment. Considerable redundancy and internal testing of the handling of signals by the computer program will be necessary so that error-free function over months or a year is achieved.

2. Monitoring-Self-testing methods - In spite of reasons given in previous discussions which indicate the potential for stability at the level of 10^{-12}, the design of this experiment needs methods to test for it continually, periodically, or at least before and after each run. A comparison of the four signals provides one such test.

If the system symmetry is such that the four signals actually represent a balance in which differences of as much as 10^{-6} are present in the gravitational torques, then changes in one of the q's or in

Q are only read relatively. That is, any hope is lost of absolute readings at the 10^{-12} level. This is, however, consistent with the basic philosophy of this experiment.

As one means of introducing test signals a test mass, occasionally located at some standard position, would introduce alternating signals at twice the frequency of the rotation of the table. These would, preferably, be at levels greater than those of the expected signals and therefore dominate noise by large factors. Any alternating signal normally present due to imbalance in the static mass distribution about the experiment would have to be subtracted.

The isolation effect of off-resonance driving of the small masses, as discussed previously means that a rather large test mass would be needed. Its location should be such that it has a symmetrical effect on all four small mass systems or at least its location must be precisely repeatable. Constancy of the magnitude of effect from time to time would be one test that the small masses and their lever arms were not changing, their restoring torques were constant, and elements in the signal chain were not changing. Uniformity of the effect among the four signals would be another test.

One alternative testing method would be the introduction of specified test impulses in the feedback loop, with the comparison of responses from time to time and between small masses. Other variations of this method are being considered such as mechanical impulses (e.g., pulses of laser light) introduced into the feedback loop.

These schemes do not test for variation of the

large masses M, or their lever arms ℓ_1. The consistency of four independent signals from the small mass systems would be evidence of their stability and hence, only indirectly imply the stability of the large-mass system.

3. Cooling and refrigeration - Since the experiment is to be run at 50 mK it will require a ^{3}He dilution refrigerator. The design of experiments of this size and larger to operate at this and lower temperatures has been carried out in the past.[21]

Some of the special low-temperature problems in this experiment are the following: 1) The makeup feed of liquid helium to the rotating system must be accomplished without contributing vibrations to the system. 2) Boiloff must be limited to locations where it will not add vibrations to the isolated part of the system. 3) The temperature gradient must be kept small. 4) Magnetic fields within the shield must have almost no time variation. 5) Electrostatic shielding and interconnections must be complete within the shield. 6) If the rotations are to be driven from a room temperature position, a cooled, rotating seal must be designed. (Our present inclination is to avoid this by designing the complete rotation system within the shield.) 7) Thermal connection is needed (to the small masses) of a kind which will not interfere with the basic sensitivity of the system.

4. Shapes of masses - Although spherical masses have been assumed in previous sections of this paper, this is not meant to imply that it is a settled question. Other mass shapes could be used to enhance the basic sensitivity of the balance. For example, in the limit of spherical small masses and planar large masses there is no dependence on distance, hence angle. Thus $C = \partial Tq/\partial\theta = 0$ and there is an infinite sensitivity.

Other problems are present in an experiment such as this, such as design of the mass supports (particularly if high symmetry is used) and a method of powering the experiment reliably over long periods of time.

Clearly, the experiment is a difficult one, as many fundamental experiments are. Nevertheless, the importance of the question, and the need for confidence such as can be found in a laboratory experiment give a high incentive to doing the experiment.

Acknowledgements

A number of the ideas in this experiment have come from communications with others. We wish particularly to acknowledge discussions with William Fairbank, Stanford; Richard Deslattes, N.B.S.; Robert Dicke, Princeton; and William Hamilton, LSU.

Notes

1. Charles W. Misner, Kip S. Thorne, and John Archibald Wheeler, Gravitation (San Francisco: W. H. Freeman and Company, 1973), p. 1123.

Discussion

Question: How precisely can one determine the angle?

Dr. Ritter: The angle can be determined better than one part in 10^{13}. Positional accuracies were obtained by the group of Fairbanks up to 10^{-17} cm.

Question: What does an accuracy of position less than the nuclear radius mean?

Dr. Ritter: Well, because it's an average over a lot of nuclei. The same way as the eardrum notices sounds that correspond to displacements of a small fraction of atomic diameters, if the signal is coherent.

2. Jesse W. Beams, *Physics Today 24*, (5) (1971): 35; R. D. Rose et al. *Phys. Rev. Lett. 23*, (1969): 655.

3. Ian Roxburgh, Fifth "Cambridge" Conference on Relativity, June 10, 1974, and private communication.

4. C. Brans and R. H. Dicke, *Phys. Rev. 124*, (1961): 925; R. H. Dicke, *Am. J. Phys. 31*, (1973): 500.

5. J. P. Turneaure and S. R. Stein, *Atomic Masses and Fundamental Constants 5*, ed. J. H. Sanders and A. H. Wapstra (New York: Plenum Press, 1976) p. 636.

6. G. G. Luther, W. R. Towler, R. D. Deslattes, R. Lowry and J. Beams, in *Atomic Masses and Fundamental Constants 5*, ed. J. H. Sanders and A. H. Wapstra (New York: Plenum Press, 1976) p. 592; W. R. Towler, private communication.

7. Roll, P. G., Krotkov, R. and Dicke, R. H., *Annals of Physics 26*, (1964): 442.

8. McCombie, C. W., *Rept. Prog. in Phys. 16*, (1953): 266.

9. Hugh D. Young, *Statistical Treatment of Experimental Data* (New York: McGraw-Hill Book Company, Inc., 1962) p. 122.

10. A. Sakuma, Bureau International des Poids et Mesures, Sevre, private communication.

11. W. M. Fairbank, Stanford, private communication.

12. Richard Deslattes, National Bureau of Standards, Gaithersburg, private communication.

13. W. M. Fairbank et al. in *Experimental Gravitation*, LVI Corso, ed. Bertotti, (New York: Academic Press, 1973) p. 294; P. W. Worden, Jr. and C.W.F. Everitt, in *Experimental Gravitation*, LVI Corso, ed. Bertotti, (New York: Academic Press, 1973) p. 381.

14. R. P. Giffard, R. A. Webb and J. C. Wheatley, *Jour. Low Temp. Phys. 6*, (1972): 533; J. E. Zimmerman, *Cryogenics 12*, (1972): 19.

15. W. M. Fairbank, in *Experimental Gravitation*, LVI Corso, ed. Bertotti (New York: Academic Press, 1973) p. 280 and references cited.

16. W. M. Fairbank, F. C. Witteborn, J.M.J. Madey and J. M. Lockhart, in *Experimental Gravitation*, LVI Corso, ed. B. Bertotti (New York: Academic Press, 1973) p. 310.

17. Samuel A. Schaaf and Paul L. Chambre, *Flow of Rarefied Gases*, Princeton Aeronautical Paperback No. 8, ed. C. du P. Donaldson, (Princeton University Press, 1961);

Harold S. Morton, Jr., University of Virginia, private communication.

18. Ref. 10; and A. Sakuma, in *Precision Measurement and Fundamental Constants*, ed. D. N. Langenberg and B. H. Taylor (U. S. Dept. of Commerce, NBS Publ. #343, Aug. 1971) p. 447.

19. George Gillies, "Development of a Drive System Yielding Highly Constant Average Speed", thesis, M.Sc., University of Virginia, May 1976.

20. W. H. Munk and G.J.F. MacDonald, *The Rotation of the Earth* (Cambridge, England: Cambridge Univ. Press, 1960).

21. W. M. Fairbank et al. ed. Bertotti, p. 294; W. O. Hamilton, private communication.

A Radar Test of the Constancy of the Gravitational Interaction

by R. D. Reasenberg and I. I. Shapiro

1. *Introduction*

Speculation about a possible time variation of the coefficient of gravitational interaction, G, has persisted for about 50 years and centers mainly on the Large Numbers Hypothesis. In 1964 it became apparent that it should be possible to test certain consequences of this hypothesis using radar observations of the planets. The first published results of such an attempt (Shapiro et al., 1971) established that $|\dot{G}/G| < 4 \times 10^{-10}$/yr and contained the prediction that this bound would improve with time as $t^{5/2}$.

In this and subsequent analyses of the radar echo delays, the equations of planetary motion with constant G were replaced with otherwise identical equations in which

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